Comment: "Basic Strategy for Card Counters"

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Abstract. We raise questions about the finding in Werthamer's "Basic strategy for card counters: An analytic approach" that CBS outperforms count-dependent strategies under certain conditions.

1 Counter Basic Strategy

We have two papers in this collection that deal with the topic of Counter Basic Strategy (CBS). The Marcus paper (2007, this volume, pp. 35–46) uses simulation to determine the CBS for various games, while the Werthamer paper (2007, this volume, pp. 47–57) takes a more analytic approach. I will assume that the reader has read those papers, and is familiar with the concepts and definitions contained in them.

The Werthamer paper uses an "effective true count" γ^* to determine the counter basic strategy. Basically, a play whose index number exceeds γ^* is included in the CBS.

We can come up with a simple formula for estimating this γ^*. Let us say that we are considering a specific play, such as standing on 15 vs. 10. We start by assuming that the relative gain in expectation for standing is approximately a linear function of the true count γ of the form

$$E(\gamma) = K(\gamma - \gamma^c), \qquad (1)$$

where K is a positive constant, and γ^c is the critical index for this play. That is, we would stand when $\gamma > \gamma^c$ and hit when $\gamma < \gamma^c$. Note that the specific values of K and γ^c vary from play to play; however we are assuming that each play does admit a linear approximation of the form above.

Now suppose we are playing a specific game of blackjack, with a specific betting ramp. That is, we encounter a series of true counts γ_j, each occurring with probability p_j. At each true count we place a bet b_j. We consider a specific play, such as standing on 15 vs. 10, and wish to determine if our basic strategy should be "always stand" or "always hit."

The total gain G from the strategy "always stand" will be approximately

$$G \approx \sum_j p_j b_j E(\gamma_j) = K \sum_j p_j b_j (\gamma_j - \gamma^c). \qquad (2)$$

We would adopt this strategy if G is positive, which occurs when

$$\gamma^c < \frac{\sum_j p_j b_j \gamma_j}{\sum_j p_j b_j}. \qquad (3)$$

The expression on the right side of the last inequality is our estimate of γ^*. This is simply a weighted average of the true counts γ_j, weighted by the total amount of money we wager at that level. This clearly depends on the betting ramp; a stronger betting ramp with a wider spread will produce a higher γ^* and lead to more plays being included in our CBS. This result is borne out in the Marcus paper, where we see plays with higher index numbers becoming included in the CBS.

There are two assumptions made in the previous derivation. One is that the expectation is a linear function of true count. Griffin made extensive use of this assumption in his work, and it is reasonably accurate for the modest true counts that are involved in typical blackjack play.

However, there is another assumption that is more problematic. We have been implicitly assuming that the true count at the time we make our playing decision is the same as the true count when we place our bets. I will refer to the latter as the pre-deal count, and to the former as the playing count.

To illustrate the effects of this assumption, let us consider a specific playing decision: whether or not to double 8 vs. 6. We have seen (at least) three cards since we have made our bet: the two cards in our 8, and the dealer's 6. These cards are all low cards, and in any reasonable count system our running count would have increased several points. With the High-Low system, it would have gone up 3 points.

The effect this has on the true count depends on how many cards remain unseen. If we are half way through a 6-deck shoe, then our true count would have gone up 1 point per deck. If we have 1 deck remaining, it would be 3 points per deck. If we are in a deeply-dealt single-deck game and only a quarter-deck is left, then our true count would have risen by 12 points per deck! Obviously the assumption that the pre-deal and playing counts are the same becomes more problematic with deeper penetration. Werthamer used this assumption to produce his count-dependent strategy (CDS), and he does note that it results in an under-estimation of the gain from CDS.

Werthamer makes a rather astonishing claim on behalf of his CBS. He states that his CBS outperforms the CDS in the games that he analyzed. On its face, this result does not seem possible. Any fixed strategy, such as CBS is a particular Count-Dependent system, with indices equal to the extreme values. For example, in the count system used here, the true count is always in the interval of $[-1.52, 1.28]$ points per card, or $[-67, 80]$ points per deck. His strategy of "never" taking insurance is equivalent to a count-dependent player using an index of +80. Similarly his strategy of "always" doubling A9 vs. 5 is equivalent to using an index of −67 for this play. The optimal count-dependent system must necessarily be better. This may suggest that the author has done a poor job of computing the indices for his count-dependent system; it does not show the superiority of his CBS.

Werthamer attempts to explain this "paradox" by stating:

> One explanation for this paradox is that Count-Dependent maximizes the expected return at each true count, but at zero depth; it is not recalculated for each separate depth value. In contrast, CBS maximizes the yield, the dominant contributions to which come from sizable depths, where the true count is more likely to take on large, positive values and the expected returns and bet sizes are correspondingly larger. The depth dependence of the expected return and play strategy, although almost always ignored in the blackjack literature, is a significant influence on our computational results.

I do not agree with this. First, modern techniques for generating playing indices for Count-Dependent systems do use simulated data taken throughout the entire shoe, and not just from the first round. Second, the fact that high true counts occur deep in the deck does nothing to suggest that −67 would be the best possible index. An index of 0 would play those "large positive counts" as well as −67, and play some other hands better.

Also, the effect of depth on strategy has been studied and it has not been found to be significant, for true-counted systems. This may be one of the reasons why the effect is "largely ignored." I will stipulate that a CDS for an 80% single-deck game will be different than the CDS for a 20% game, However, the differences will be minor, with some indices moving by a point or so. Using a system where the index numbers are off by a point is not nearly as costly as using no index number. That is, a CDS that is tailored to a different level is going to be better than using a CBS.

2 Simulation results

In order to further examine this issue, I asked Norm Wattenberger to carry out simulations of it. Wattenberger is the author and producer of the *Casino Vérité* line of blackjack software products, which enjoy a well-deserved reputation with the card-counting community as representing the "gold standard" in blackjack software.

The count system that Werthamer used in his paper was an "ultimate" point count using decimal values. It is equivalent to a 128-level integer system. There are practical problems in simulating this system. However, there is a 4-level system listed on page 45 of Griffin (1999) that has a 99.6% correlation with this ultimate count. We decided to use this system. It has tag values $\langle -4, 2, 3, 3, 4, 3, 2, 0, -1, -3 \rangle$.

First, Wattenberger generated a set of single-deck index numbers for this system for the “Sweet 16” plays. These are the same plays that are in the “Illustrious 18,” except that the two ten-splits are not included. That is, it will perform somewhat worse than the full I-18. This comprised our CDS. Then he produced a battery of simulations comparing the CDS, the Werthamer CBS, and conventional BS with various bet spreads. He used the widely accepted SCORE parameter to measure the effectiveness of each system. (SCORE is an acronym for Systematic Comparison of Risk and Expectation. It measures the total return for each system with the risk of ruin held constant.) In each case, the betting schedule was optimized for each particular system, so as to produce the highest possible SCORE. The rules are the same as in the paper: single deck, S17, no doubling after split, and no re-splitting of pairs. Spreads of 1–4, 1–8, and 1–12 were employed. For each spread, there was a simulation with 1 player and with 4 players.

The results of the simulations are shown in Figures 1–3.

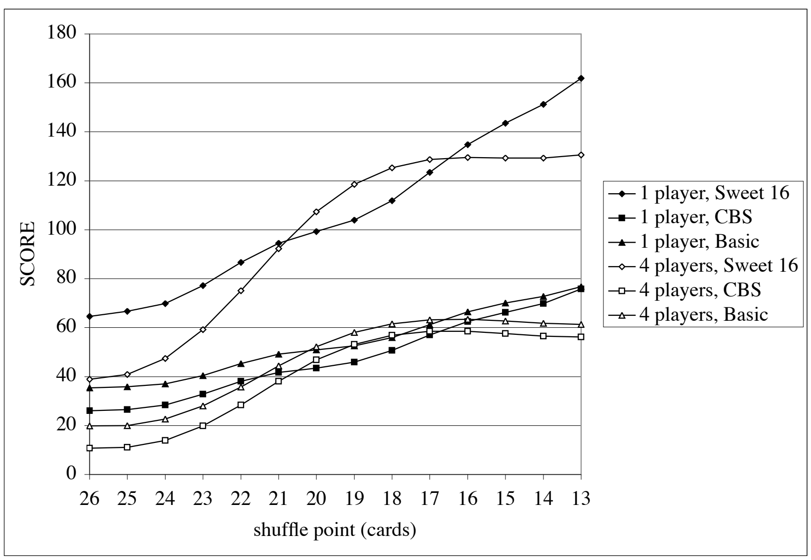

Fig. 1. SCORE as a function of shuffle point when the spread is 1–4.

For the 1–4 spread, the CBS is actually worse than BS. This should not be surprising because the CBS was constructed for a much wider spread (1–10). However, even at the wider spreads we see that it under-performs relative to our simple CDS. In the 1–8 games it only achieved 62% of the CDS SCORE in the head-on game, and a little less in the 4-player one. Even in the 1–12 game, it never gets more than 68%. In short, it isn’t even close.

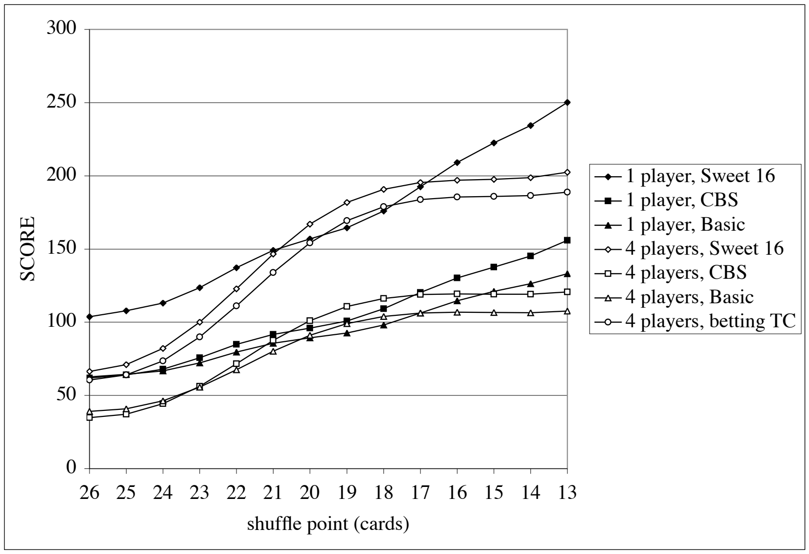

Fig. 2. SCORE as a function of shuffle point when the spread is 1–8.

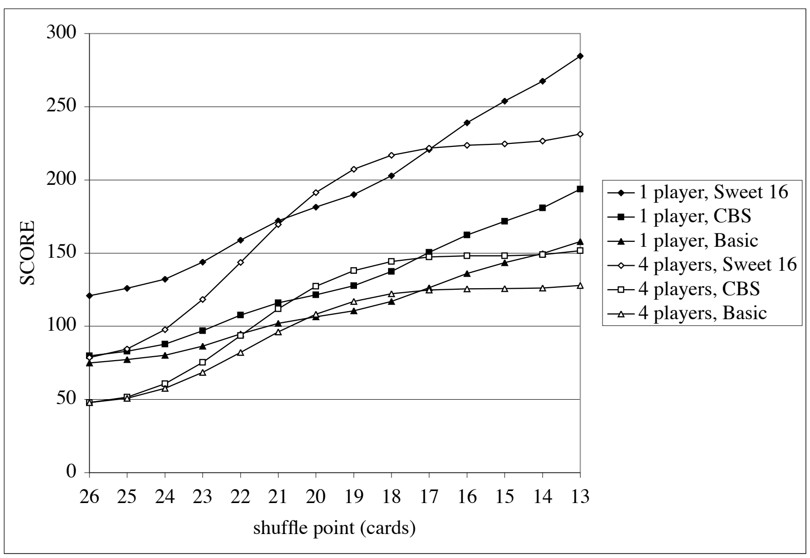

Fig. 3. SCORE as a function of shuffle point when the spread is 1–12.

I originally thought that Werthamer's CDS performed so poorly because it was based on the pre-deal count, and not on the playing count. However, Wattenberger did one more sim to consider this effect. Here the same CDS was used, but the decision had to be made without considering the cards seen during the round. As expected, it did perform worse than the full CDS but was still significantly better than the CBS. Apparently, there must be other features in the software that Werthamer used that led to his conclusion.

This is just one set of simulations comparing one CDS with one CBS. However, I would strongly urge any player who is considering scrapping his/her CDS in favor of some new CBS, to perform his/her own simulation under his/her own conditions and compare the results. If the CDS does perform worse than the CBS, then I would suggest looking for errors in that CDS.

Having just said that a CBS cannot outperform a reasonable CDS, let me add that there are other good reasons to consider CBS. Let me conclude by discussing some positive applications for it.

Many counters use only a limited number of index numbers, such as the I-18, and employ a fixed strategy for the other plays. Typically, they use the standard BS for this, but I will suggest that they would do better to use a CBS for those plays. One problem with the CBS is that it varies depending on the bet ramp; this is seen in the Marcus data and in our discussion of γ^*. However, it is very hard to construct a reasonable bet spread that has a γ^* less than 2 High-Low points per deck, or its equivalent. This suggests that those plays would be CBS for any "reasonable" ramp. These would include standing on 16 vs. 10, 12 vs. 3, A7 vs. A, and doubling 9 vs. 2, 11 vs. A, A8 vs. 5 or 6, and A3 vs. 4. (Some of these plays are already BS in single-deck games, but none of them are BS in S17 6-deck games.)

What about players who do not want to learn 18 indices, but are willing to learn a few? Conventional wisdom might suggest that they learn such plays as 16 vs. 10, 12 vs. 3, and 9 vs. 2. I am suggesting instead that they use a CBS for those plays, and instead focus the index numbers for insurance, 15 vs. 10, 12 vs. 2, 12 vs. 2, 9 vs. 7, and 10 vs. A.

Finally there is the issue of special rules and promotions. Occasionally a casino will offer a nonstandard rule, sometimes as a special promotion and sometimes on a regular basis. For example, I have seen jokers used with various rules in some circumstances; special payouts on some 5-card hands, even a bonus for breaking with 26. Some of the tribal casinos in the Midwest have a special rule that allow a player to draw a third card to split Aces by doubling down.

Sometimes these rules result in strategy changes. Typically, we compute the strategy for the full 52-card or 312-card pack. That is, a Basic Strategy. Some of these plays have index numbers, but it may not be practical to memorize a whole set of numbers for a promotion that will last only a limited time. I am suggesting that if index numbers are not going to be used, then the strategy developed should be a CBS, with more weight given to the higher true counts where more money will be bet.

References

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2. Marcus, Hal I. (2007) New blackjack strategy for players who modify their bets based on the count. In: Ethier, Stewart N. and Eadington, William R. (eds.) *Optimal Play: Mathematical Studies of Games and Gambling*. Institute for the Study of Gambling and Commercial Gaming, University of Nevada, Reno, 35–46.
3. Werthamer, N. Richard (2007) Basic strategy for card counters: An analytic approach. In: Ethier, Stewart N. and Eadington, William R. (eds.) *Optimal Play: Mathematical Studies of Games and Gambling*. Institute for the Study of Gambling and Commercial Gaming, University of Nevada, Reno, 47–57.